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Strings from tachyons: the $c = 1$ matrix reloaded

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Abstract: We propose a new interpretation of the $c = 1$ matrix model as the world-line theory of N unstable D-particles, in which the hermitian matrix is provided by the non-abelian open string tachyon. For D-branes in 1+1-d string theory, we find a direct quantitative match between the closed string emission due to a rolling tachyon and that due to a rolling eigenvalue in the matrix model. We explain the origin of the double-scaling limit, and interpret it as an extreme representative of a large equivalence class of dual theories. Finally, we define a concrete decoupling limit of unstable D-particles in IIB string theory that reduces to the $c = 1$ matrix model, suggesting that 1+1-d string theory represents the near-horizon limit of an ultra-dense gas of IIB D-particles.

Keywords: Matrix Models, Tachyon Condensation, D-branes.

Contents

1. Introduction

The duality between open and closed string theory has led to various deep and surprising insights into the fundamental structure of both systems. A relatively recent, but still incompletely understood, example of such a duality is the observation that unstable D-branes can completely decay into closed strings via open string tachyon condensation [1, 2]. Both the full time evolution as well as the final stage of this decay process are fascinating arenas for further study [3]–[7]. In particular, it would be desirable to find a controllable description of closed string creation from open string tachyon matter [8, 9, 10].

It is a natural strategy to try to apply the lessons of other open/closed string dualities, like Matrix theory and the AdS/CFT correspondence, to this problem. In particular, we could attempt to find a regime in which the tachyon degrees of freedom are naively decoupled from the bulk closed strings, but at the same time become fully equivalent to a complete closed string theory in an appropriate near horizon geometry. Experience tells us that this can be achieved if we can take a suitable large-N limit and tune parameters, such that the tachyon matter becomes ultra-light and saturates all possible degrees of freedom of the theory.

With this motivation, we will consider in this paper the non-abelian tachyon dynamics of a dense gas of many unstable D-particles. To enable investigation of the tachyon mode in isolation, we will consider special models, or regimes of couplings, in which all other degrees of freedom of the D-particles, such as their space-time positions, are either absent or decoupled. A specific string model with this property is 1+1-dimensional bosonic string theory, which has a well-known dual description in terms of matrix quantum mechanics; for reviews see e.g. [11]–[14].

This duality between 1+1-d strings and $c = 1$ matrix quantum mechanics is the oldest known example of a holographic equivalence, and has several attractive features in comparison with the examples found later. It is a holographic theory with an S-matrix description, and therefore more similar to a model of holography in flat space. Secondly, both sides of the duality have overlapping weakly coupled regimes; the matrix model is even exactly soluble. This allows very precise quantitative comparisons. The duality also has various mysterious features and unresolved puzzles [15].

In the light of more recent developments, it is natural to suspect that the $c = 1$ matrix degree of freedom should be related to the open string tachyon of unstable D-particles of the 1+1-d string theory itself.[1] We will present concrete evidence in support of this identification. In particular we will find a direct quantitative match between the closed string emission due to a rolling tachyon and that due to a rolling eigenvalue in the matrix model. Via this new interpretation of the matrix model, we will be able to clarify several of its somewhat mysterious features. We will explain the physical meaning of the double-scaling limit as a decoupling limit, and interpret it as selecting an extreme representative of a large equivalence class of dual theories in which D-branes are replaced by their back-reaction on the closed string background. The projection onto singlet states is naturally implemented by the worldline gauge invariance. The D-brane perspective also sheds new light on the non-perturbative instability of the matrix model against tunneling of eigenvalues towards the wrong side of the potential barrier: it corresponds to decay of the open string tachyon towards the regime where its potential is unbounded from below. Due to this instability, it would appear that 1+1-d string theory is an incomplete model, since it does not seem to have a completely self-consistent non-perturbative definition.

It is an important question, therefore, whether it is possible to obtain 1+1-d string theory via a special limit of one of the consistent supersymmetric string theories. In the last section, we will propose a confirmative answer to this question, by defining a natural decoupling limit of a dense collection of unstable D-particles in IIB string theory, in which the world-line theory reduces to the $c = 1$ matrix model. This correspondence suggests that 1+1-d string theory can be given its rightful place within the world of consistent theories, as the near-horizon limit of a dense cluster of unstable D-particles.

2. Rolling and bouncing tachyons

We begin with a brief summary of some recent results and insights about open string tachyon dynamics on non-supersymmetric D-branes, that will be useful for our later discussion. We will restrict our attention to the case of unstable D-particles, both in bosonic and supersymmetric string theory.

[1] Indeed, a comment to this effect appears in [16].

The worldline theory of a D-particle in bosonic string theory is a quantum mechanical system, with one unstable tachyonic degree of freedom T. Its equation of motion is the requirement that the corresponding worldsheet boundary interaction

$$S_{\text{open}} = \int d\xi \; T(X^0(\xi)) \tag{2.1}$$

defines a proper boundary conformal field theory. In case the closed string background is static, so that

$$S_{\text{closed}} = S_{CFT} + \frac{1}{4\pi}\int d^2\sigma\, (\partial_\alpha X^0)^2\,, \tag{2.2}$$

where S_{CFT} is any $c = 25$ CFT describing the spatial directions of the target space, the following trajectories

$$T_{\text{roll}}(X^0) \;=\; \lambda \exp X^0 \tag{2.3}$$

$$T_{\text{bounce}}(X^0) = \lambda \cosh X^0 \tag{2.4}$$

are exact solutions for all values of $\lambda \leq 1/2$. The first trajectory describes a rolling solution: the tachyon starts at the top of the potential and rolls towards the minimum of its potential at $T = \infty$. The second trajectory represents a bounce: the tachyon starts from and returns to the minimum at $T = \infty$, reaching its smallest value $T = \lambda$ at $X^0 = 0$. In the interacting string theory, this bouncing tachyon solution can be thought of as being initiated at early times by a collision of closed string matter, creating an unstable D-particle which, after a finite lifetime of order $|\log\lambda|$ (for small λ), decays back into closed strings. The critical configuration with $\lambda = 1/2$ has the special feature that the boundary state formally vanishes [3]; it can be shown to be equivalent to a specific time-dependent classical closed string background [8]. This equivalence can be thought of as a perturbative (or infinitesimal) version of open/closed string duality, since it indicates that a D-particle with a particular tachyon profile can be completely absorbed via an adjustment of the closed string background. This surprising result will be a key element in our later argumentation.

The classical behavior (2.4) of the tachyon mode can be reproduced in terms of an effective mechanical model, with a Born-Infeld type lagrangian[2]

$$S(T) = -\int dt\, V(T)\sqrt{1-\dot{T}^2} \tag{2.5}$$

where

$$V(T) = \frac{1}{g_s \cosh(T/2)}\,. \tag{2.6}$$

This form of $V(T)$ applies for positive T only: for negative T, the bosonic string tachyon potential is known to be unbounded from below. We will for the most part restrict our attention to the stable region $T > 0$. In the following section we will use the matrix generalization of this effective model as our starting point for studying the quantum mechanics of the matrix valued tachyon associated with systems of many unstable D-particles. The

[2]Following [9, 17] we have performed a field redefinition relative to (2.4); the variable T appearing below has the solution $\sinh^2 T/2 = \lambda\, \cosh t$.

detailed analytic form of the potential $V(T)$ will not be essential, except for the two global properties that (i) it has a single maximum at $T = 0$, near which it behaves as

$$V(T) \simeq \frac{1}{g_s}\left(1 - \frac{1}{8}T^2 + \cdots\right), \tag{2.7}$$

and (ii) $V(T)$ exponentially decays to zero at $T \to \infty$.

In a general closed string background, the unstable D-particle has many more light degrees of freedom than just T; in particular it has coordinates $X^i(t)$ that parametrize the space-time motion of the D-particle. To enable investigation of the tachyon mode in isolation, we will consider special models, or regimes of couplings, in which these other degrees of freedom are either absent or decoupled. A specific bosonic string model which is known to satisfy this property is the 1-dimensional non-critical string, or equivalently, the 1+1-dimensional critical string theory, to which we now turn.

3. D-branes in 1+1-d string theory

The space-like motion of the 1+1-d critical string is described by the Liouville conformal field theory

$$S_{\rm bulk} = \frac{1}{4\pi}\int d^2\sigma\,\left((\partial_a\varphi)^2 + QR^{(2)}\varphi + 4\pi\mu e^{2b\varphi}\right), \tag{3.1}$$

with $Q = b + \frac{1}{b}$ and central charge $c_L = 1 + 6Q^2$. The case of interest to us is $c_L = 25,\ Q = 2,\ b = 1$, but it will sometimes be useful to keep $b \neq 1$ as a regulator. The $c_L = 25$ Liouville CFT represents a classical string background of the effective 1+1-d target space-time field theory (here $\mathcal{T}$ denotes the *closed* string tachyon, and R the 1+1-d target-space curvature scalar)

$$S_{\rm eff} = \int d^2x\,\sqrt{-G}e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 - (\nabla\mathcal{T})^2 + 4\mathcal{T}^2 + 16 + \cdots\right). \tag{3.2}$$

Besides the standard classical tachyon profile $\mathcal{T} \simeq e^{2\varphi}$, this action also admits $\mathcal{T} \simeq \varphi e^{2\varphi}$ as a static solution for the closed string tachyon, and the latter solution dominates for large negative φ. The $c = 1$ string background

$$\Phi(\varphi) \simeq 2\varphi\,, \qquad \mathcal{T}(\varphi) \simeq \left(\varphi + \frac{1}{2}\log\mu\right)\mu e^{2\varphi} \tag{3.3}$$

is characterized by just one single parameter, which we view as related to the string coupling $g_s = e^{\Phi}$ at the location of the "tachyon wall" (i.e. the place where $\mathcal{T}$ becomes of order 1 in string units) via

$$g_{\rm eff} \simeq \frac{1}{\mu}\,, \tag{3.4}$$

or as related to the value of $\mathcal{T}$ at the location $\varphi = 0$ of the "dilaton wall" (i.e. where the string coupling is of order 1) via

$$\mathcal{T}(0) \simeq \frac{\mu}{2}\log\mu\,. \tag{3.5}$$

The action (3.2) has only one single propagating degree of freedom, which we can take to be the closed string tachyon $\mathcal{T}$. In spite of its name, it in fact satisfies — due to the presence of the linear dilaton — a massless wave-equation. Vertex operators corresponding to normalized states look like

$$V_P = e^{(Q+iP)\varphi} \tag{3.6}$$

with P real and positive.[3] V_P has conformal dimension $\Delta_P = \frac{1}{2}(Q^2+P^2)$. We will call the state with this momentum $|v_P\rangle$, and take it to be normalized so that $\langle v_{P'}|v_P\rangle = \pi\delta(P-P')$.

The possible consistent boundary conditions of Liouville CFT have recently been studied in [20, 21] (see also [22, 23, 24]). This open Liouville theory is defined by introducing the Weyl-invariant boundary interaction

$$S_{\text{bdy}} = \frac{1}{4\pi}\int_{\partial\Gamma}\left(\frac{QK}{2\pi}\varphi + \mu_B e^{b\varphi}\right)d\xi\,, \tag{3.7}$$

where K is the extrinsic curvature, ξ is a coordinate on the boundary, and μ_B is the boundary cosmological constant. It represents a continuous marginal coupling of the boundary CFT. [4]

An interesting quantity is the overlap of the momentum eigenstate $|v_P\rangle$ with the boundary state corresponding to the boundary action described above. It corresponds to the one-point function of the vertex operator V_P on the disk. It is given by [20]

$$\langle v_P|B_s\rangle = \frac{\hat{c}}{iP}(\pi\mu\gamma(b^2))^{-iP/2b}\Gamma(1+ibP)\Gamma\left(1+\frac{iP}{b}\right)\cos(\pi sP)\,, \tag{3.8}$$

with $\hat{c}$ an overall normalization constant, $\gamma(x) \equiv \Gamma(x)/\Gamma(1-x)$ and s is a parameter[5] related to μ_B by the relation

$$\cosh^2\pi bs = \frac{\mu_B^2}{\mu}\sin\pi b^2\,. \tag{3.9}$$

The boundary state with label s is identical to that with label $-s$. This explicit expression for of $\langle v_P|B_s\rangle$ was found in [20] by deriving a functional equation that it must satisfy; the parameter s appearing in (3.9) arises in solving this equation. Note that, unlike for D-branes in flat space, the overlap (3.8) has a quite non-trivial P dependence and is not just a phase factor. Specializing to the critical value of $b = 1$ gives

$$\langle v_P|B_s\rangle = c\,e^{-i\delta(P)}\,\frac{\pi\cos(\pi sP)}{\sinh(\pi P)} \tag{3.10}$$

where

$$e^{-i\delta(P)} = (\pi\mu)^{-iP/2}\frac{\Gamma(1+iP)}{\Gamma(1-iP)}\,. \tag{3.11}$$

[3]These in fact do not correspond to good local operators, since the solution to the classical Liouville equation in their presence implies a hyperbolic metric (i.e. a throat) in their neighborhood [18, 19].

[4]We would like to thank J. Teschner for correcting an error in an earlier version of this paper.

[5]Note the slight change of conventions:

$$s = s_{FZZ} = 2s_T\,, \qquad P = 2P_{FZZ} = 2P_T$$

where the subscript FZZ labels quantities appearing in [20] and the subscript T labels those in [21].

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In the $b \to 1$ limit, the right hand side of (3.9) remains finite. However, a particularly interesting case arises when $\mu_B = 0$, which implies that

$$\cosh(\pi s) = 0 \,. \tag{3.12}$$

This is solved when s takes one of a discrete set of imaginary values

$$s = \frac{i}{2}(2m+1)\,, \qquad m \in \mathbf{Z}\,. \tag{3.13}$$

Investigation of the spectrum of open string states associated with each boundary state $|B_s\rangle$ furthermore reveals that as one increases n one finds increasingly tachyonic open-string modes [21]. We will focus on the minimal value $s = \frac{i}{2}$. Note that these D-objects do not have a continuous degree of freedom corresponding to their space-like position.

To obtain a bit more insight into the structure of the boundary state $|B_s\rangle$ it is instructive to introduce a boundary state with fixed *length* equal to ℓ via

$$|B_s\rangle = \int \frac{d\ell}{\ell}\, e^{-\ell\sqrt{\mu}\cosh(\pi s)}\, |W(\ell)\rangle\,. \tag{3.14}$$

The overlap of this new boundary state $|W(\ell)\rangle$ with the momentum eigenstate $|v_P\rangle$ takes the form

$$W(\ell, P) \equiv \langle v_P | W(\ell)\rangle = c\, e^{-i\delta(P)}\, P\, K_{iP}(\sqrt{\mu}\ell)\,. \tag{3.15}$$

Interestingly, this expression satisfies the Schrodinger equation

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial\phi_0^2} + 2\pi\mu e^{2\phi_0} - \frac{1}{2}P^2\right) W(\ell, P) = 0 \qquad \ell = e^{\phi_0} \tag{3.16}$$

which is often referred to as the "mini-superspace Wheeler-De Witt equation" of the 2-d Liouville gravity theory, since it takes the form of the zero-mode truncation of the Liouville equation of motion. It shows that $W(\ell, P)$ can be thought of as the space-time profile of the tachyon mode created by the microscopic vertex operator V_P of momentum P. Applying the Laplace transformation (3.14) to the result (3.15) reproduces (3.10), via the identity

$$\int \frac{d\ell}{\ell}\; e^{-\ell\sqrt{\mu}\cosh(\pi s)} K_{iP}(\sqrt{\mu}\,\ell) = \frac{\cos(\pi P s)}{P\sinh(\pi P)}\,. \tag{3.17}$$

Another very interesting class of boundary conditions for the Liouville theory were studied in [25]. These boundary states have the property that the only state that propagates in the open string channel is the identity operator. They are highly localized in the Liouville direction, and do not have a space-like position. As pointed out in [16], this implies that their worldline description reduces to a matrix model.[6]

In 1+1-d string theory, the Liouville theory is supplemented with the $c = 1$ CFT of the time coordinate X^0 and we can thus consider boundary states that describe a D-particle with a rolling open string tachyon on its world-line. When the Liouville boundary state

[6]A study of their interpretation as D-objects, and their description in the matrix model was recently performed in [26, 27].

has $\mu_B = 0$, the profile of the open-string mode arises purely from the X^0 boundary state; otherwise, there is a time-independent term in the profile. This is our main motivation for focussing on $\mu_B = 0$. For later comparison with the $c = 1$ matrix model, we would like to determine the one-point function that expresses the leading order emission of closed string tachyons from this boundary state. For concreteness, we will consider the half-brane trajectory (2.3). The corresponding boundary state takes the form of the tensor product

$$|B\rangle = |B_\lambda\rangle \otimes |B_s\rangle\,. \tag{3.18}$$

We are interested in the overlap of this boundary state with the state $|v_\omega\rangle \otimes |v_P\rangle$ created by the vertex operator $V_{\omega,P} = e^{i\omega X^0 + (Q+iP)\varphi}$ of given energy ω and Liouville momentum P, subject to the mass-shell condition that $\omega = |P|$. The time-dependent state $|B_\lambda\rangle$ has a non-zero overlap with energy eigenstate $|v_\omega\rangle$ — normalized according to $\langle v_\omega | v_{\omega'}\rangle = \pi\delta(\omega - \omega')$) equal to

$$\langle v_\omega | B_\lambda \rangle = \lambda^{-i\omega}\frac{\pi}{\sinh \pi\omega}\,. \tag{3.19}$$

So the total production amplitude is (specializing to $s = \pm i/2$)

$$\mathcal{A}(\omega, P) = \langle v_\omega | B_\lambda\rangle \langle v_P | B_s\rangle = \frac{c\,\pi^2 e^{-i\delta(P)}\lambda^{-i\omega}}{\sinh(\pi P/2)\sinh(\pi\omega)}\,, \qquad \omega = |P|\,. \tag{3.20}$$

In a section five we will reproduce this exact same amplitude by considering the emission due to a classical rolling eigenvalue in the $c = 1$ matrix model.

4. D-particle gas in a 1-D box

We will now proceed to analyze the quantum mechanics of the non-abelian tachyon mode that lives on the worldline of (a bound state of) many unstable D-particles. That we can treat the tachyon mode in isolation, without coupling to other worldvolume fields, will be justified for unstable D-particles in the bosonic and type-IIB theories in section 7. In the 1+1-d theory, there are indeed D-branes with this property.[7] We will assume that the effective action of the T mode is as given in eqn (2.5).

First, however, let us address an apparent puzzle. Perturbative study (for small values of the Liouville interaction $\mu e^{2\phi}$) of the open 1+1-d string spectrum reveals that, like its closed string cousin, the open string tachyon in 1+1-d string theory is not really "tachyonic" but rather "massless": its perturbative potential starts out flat, with zero second derivative instead of with a negative one as in eqn (2.7). Indeed, in general the presence of open and closed string tachyonic modes can be seen as a consequence of the Hagedorn growth of the number massive string states; the 1+1-d model has no massive on-shell degrees of freedom, and its physical spectrum is therefore free of tachyonic instabilities. In other words, in the perturbative regime of small $\mu e^{2\phi}$, D-particles in 1+1-d string theory would appear to be perfectly stable! So how can the action (2.5), with the unstable potential (2.6), be a correct effective model?

[7] It has been pointed out in [16, 28], this analysis is most naturally applied to the branes of [25].

The justification for (2.5) is that its classical trajectories reproduce the consistent tachyon profiles (2.4). This reasoning is still perfectly valid in the 1+1-d theory. The implicit assumption leading to the apparent contradiction is that a D-particle can be independently localized in the Liouville direction φ as well as in T. This assumption, however, is invalidated by the fact that the type of D-particle we wish to study has only T as its low energy degree of freedom, and does not have an independent φ position. This suggests that this φ position is in fact correlated with the value of T, and it would seem a good guess that small values of T, near the top of the potential $V(T)$, describe a D-particle in the strongly coupled region near the "Liouville wall," while large values of T (near the flat region of the potential $V(T)$) correspond to φ values in the weakly coupled asymptotic region. All this is of course very reminiscent of the holographic dictionary of the AdS/CFT correspondence, and, more to the point, of its precursor: the c=1 matrix model.

The $c = 1$ matrix model describes 1+1-d string theory in terms of the quantum mechanics of a single large-N matrix. It has long been recognized, ever since the D-brane string revolution, that the eigenvalues of this large-N matrix have a likely interpretation as the positions of D-particles of a suitable type. Given the above discussion, it is natural to suspect that the appropriate D-particle is that of the 1+1-d theory itself. The effective lagrangian describing N such D-particles is

$$S(\mathbf{T}) = -\int dt \, \mathrm{Tr} \left(V(\mathbf{T}) \sqrt{1 - (D_t \mathbf{T})^2} \right) . \tag{4.1}$$

Here $\mathbf{T}$ is a an $N \times N$ hermitian matrix and $D_t = \partial_t + [A_t, \cdot]$, with A_t a $U(N)$ gauge "field". Choosing $A_t = 0$ gauge, its only role is to impose the Gauss Law which projects onto singlet states. Given the form (2.6)–(2.7) of $V(T)$, it is clear that the quantum mechanics of this model has the same large-N behavior as the $c = 1$ matrix model. In this section we will make this correspondence explicit, while recollecting some relevant facts about the $c = 1$ matrix quantum mechanics.

Following the standard $c = 1$ routine [29] we write

$$\mathbf{T} = \Omega^\dagger T \, \Omega \, , \qquad T = \mathrm{diag}(T_1, T_2, \ldots, T_N) \, . \tag{4.2}$$

Here the eigenvalues T_i parametrize the positions of the D-particles, while the $U(N)$ matrix Ω represents a pure gauge degree of freedom. Thus we may set $\Omega = 1$. Taking into account the proper jacobian, we can write the quantum mechanical wave-function of $\mathbf{T}$ as

$$\Psi(\mathbf{T}) = \Delta(T) \Psi(T) \, , \qquad \Delta(T) = \prod_{i<j} (T_i - T_j) \, . \tag{4.3}$$

Since $\Delta(T)$, the Vandermonde determinant, is completely anti-symmetric under interchange of the eigenvalues T_i, the reduced wave-function $\Psi(T)$ is completely anti-symmetric as well. The hamiltonian derived from (4.1) reduces to a sum of decoupled single particle hamiltonians

$$H = \sum_i h_i \, , \qquad h_i = \sqrt{\pi_i^2 + V(T_i)^2} \tag{4.4}$$

with $\pi_i = i \frac{\partial}{\partial T_i}$. The N D-particle matrix model thus reduces to a decoupled system of N fermions in one dimension, described by the relativistic hamiltonian (4.4).

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From the form of the potential $V(T)$ we see that the single particle hamiltonian h has a continuous spectrum of plane wave eigenstates. To resolve this continuum into a discrete spectrum, we will put the system in a 1-d box, by imposing reflecting boundary conditions at at some large (negative) value $T = T_0$. We will choose this cut-off location at

$$T_0 = 2\log g_s\,. \tag{4.5}$$

Eventually we will remove this IR cut-off taking the limit $g_s \to 0$.

The ground state of the system is obtained by filling the lowest N eigenvalues of h. We will assume that g_s, N and T_0 are tuned to that the fermi level defined by the N-th energy eigenvalue

$$\mu_{\scriptscriptstyle F} \equiv E_N \tag{4.6}$$

lies just below the top of the potential barrier

$$\mu_{\scriptscriptstyle F} \lesssim \mu_c\,, \qquad \mu_c \equiv V(0) = \frac{1}{g_s}\,. \tag{4.7}$$

Using the WKB approximation, and ignoring for now the quantum mechanical tunneling through the potential barrier, we can find the fermi level $\mu_{\scriptscriptstyle F}$ for given N via the Bohr-Sommerfeld condition

$$2\pi N = \oint dT\,\pi_{\scriptscriptstyle N}(T)\,, \qquad \pi_{\scriptscriptstyle N} = \sqrt{\mu_{\scriptscriptstyle F}^2 - V(T)^2}\,, \tag{4.8}$$

where the integral is around the closed trajectory with energy $\mu_{\scriptscriptstyle F}$. Thus

$$N = \frac{1}{\pi}\int_{T_0}^{T_F} dT\sqrt{\mu_{\scriptscriptstyle F}^2 - V(T)^2} \tag{4.9}$$

with $T_{\scriptscriptstyle F}$ the turning point at which $V(T_{\scriptscriptstyle F}) = \mu_{\scriptscriptstyle F}$.

An interesting quantity is the spectral density $\rho(\mu_{\scriptscriptstyle F}) = \frac{\partial N}{\partial \mu_{\scriptscriptstyle F}}$, which reveals a logarithmic divergence in the limit that $\mu_{\scriptscriptstyle F}$ approaches μ_c:

$$\rho(\mu_{\scriptscriptstyle F}) = \frac{1}{\pi}\int_{T_0}^{T_F} \frac{\mu_{\scriptscriptstyle F}\,dT}{\sqrt{\mu_{\scriptscriptstyle F}^2 - V(T)^2}} \simeq -\frac{2}{\pi}\log\mu\,, \tag{4.10}$$

with $\mu \equiv \mu_c - \mu_{\scriptscriptstyle F}$. Since we have set the cutoff on the eigenvalue space at $T_0 = 2\log g_s$, (4.10) gives the expected linear growth of the spectral density with the 1-d volume. Further, the $\log(\mu)$ divergence reflects the fact that the time that a particle spends near the top of the potential barrier diverges logarithmically for small μ. Correspondingly, its wave-function is sharply peaked near the turn-around point.

We thus reach the — with hindsight not entirely surprising — conclusion that the gas of D-particles in 1+1-d string theory, with chemical potential $\mu_{\scriptscriptstyle F}$ close to μ_c, displays the exact same universal behavior as the $c = 1$ matrix model. This correspondence becomes exact in the double scaling limit:

$$g_s \to 0\,, \qquad N \to \infty\,, \qquad \mu = \text{fixed}\,. \tag{4.11}$$

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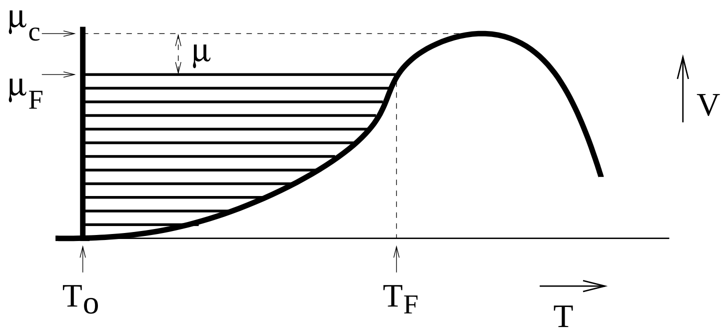

Figure 1: The tachyon potential $V(T)$ and the Fermi sea of filled energy eigenstates. The difference μ between the Fermi level $\mu_{_F}$ and the top of the potential specifies the effective string coupling of the dual 1+1-d string theory.

Essential for this correspondence is that (i) the D-particles have only one matrix coordinate T, and (ii) the effective potential $V(T)$ has a single maximum of the upside-down harmonic form (2.7).

Much work has been done to extract the scattering amplitudes from the $c = 1$ matrix model and to compare them with the corresponding string theory computations and expectations. While the duality has passed many checks, several important puzzles remain. We list of some of the main results and open questions:

(i) The target space dynamics of the matrix model can be formulated in terms of a collective field theory [30], in which the eigenvalue density ρ appears as a second quantized 1+1-d field. This collective field is expected to be related via a field redefinition to the closed string tachyon of the non-critical string theory. (In connection with this approach, it is perhaps interesting to note that the relativistic form of the eigenvalue hamiltonian (4.4) allows us to introduce a second quantized fermionic formulation based on the rather standard looking action

$$S(\Psi) = \int d^2X \left(\overline{\Psi} \left(\gamma^\mu \frac{\partial}{\partial X^\mu} - V(X^1) \right) \Psi \, + \mu_{_F} (\Psi^\dagger \Psi - N) \right) \tag{4.12}$$

where Ψ denotes a 1+1-d Majorana fermion. Collective field theory essentially arises via bosonization of the fermion Ψ.)

(ii) Exact expressions for string scattering amplitudes have been found to all orders in the effective string coupling constant

$$g_{\rm eff} = \frac{1}{\mu} \, . \tag{4.13}$$

These expressions have been checked against tree-level string theory calculations.

(ii) While the scattering amplitudes are perturbatively unitary, the matrix model reveals an inherent non-perturbative instability leading to unitarity violations of order

$e^{-2\pi\mu}$ [31]. The instability is caused by tunneling events of eigenvalues through the potential barrier. The tunneling amplitude

$$\exp\Big(-\int_{-T_F}^{T_F} dT\sqrt{V(T)^2-\mu_F^2}\,\Big) \simeq \exp(-2\pi\mu) \tag{4.14}$$

reveals the characteristic $e^{-O(1/g_{\rm eff})}$ behavior of a D-brane process. Historically, this result of course preceded [32] and in fact precipitated the later discovery of the role D-branes. With hindsight, the identification of the eigenvalue tunneling event with the D-instanton leads directly, via an application of Sen's descent relations [2], to the equivalence between eigenvalues and D-particles.

The presence of these unitarity violations turned out to be a severe problem for the $c = 1$ model, and it is as yet unclear whether these can be remedied without violating target space locality and/or causality. (For two divergent opinions on this question see [15] and [33].) This of course seriously dampened the enthusiasm about the $c = 1$ matrix model as a potential non-perturbative formalism for 1+1-d string theory: it is now considered to be an incomplete model, without any consistent and/or acceptable non-perturbative definition. Given its key role in the inception of the D-brane era, however, the model deserves a better fate than that. Our proposed identification of the matrix eigenvalues with the tachyon mode of D-particles hopefully represents a first step towards its rehabilitation. We will come back to this issue in section 7, where we propose a possible way for embedding the $c = 1$ matrix model into IIB string theory.

5. Closed-string radiation from a rolling probe-eigenvalue

The surfaces dual to matrix-model diagrams have boundaries if the matrix model is coupled to variables in the fundamental representation of the matrix quantum mechanics gauge group (This fact has been emphasized by e.g. [34, 35, 36]). Exactly such variables arise when a classical probe eigenvalue is introduced into the system, extending the rank of the $N \times N$ matrix $\mathbf{T}$ by one extra row and column

$$\mathbf{T}_{N+1\times N+1} = \begin{pmatrix} z & v_1 & v_2 & \dots \\ v_1^* & T_{11} & T_{21} & \dots \\ v_2^* & T_{12} & T_{22} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}. \tag{5.1}$$

For fixed extra eigenvalue z, the variables v_i have single line propagators that delineate a boundary of the string world-sheet, with a Dirichlet boundary condition located at z. As a quantitative test of this identification between eigenvalues of the $c = 1$ matrix and D-particles in 2D string theory, we will compute the closed string radiation produced by a classical rolling extra eigenvalue in first-order time-dependent perturbation theory. For the probe trajectory we will take

$$z(t) = \lambda\, e^t\,. \tag{5.2}$$

The corresponding perturbation hamiltonian arises as follows.

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Suppose we start from the $N+1 \times N+1$ matrix model. We can write its wave-function in terms of its eigenvalues and after splitting off the Vandermonde determinant obtain the free hamiltonian $H_{N+1} = H_0 + H_{probe}$. Here H_0 is the free hamiltonian of the $N \times N$ model and H_{probe} that of the extra eigenvalue. We want to make the probe follow a classical trajectory. We should therefore consider a wave-function of the N quantum mechanical eigenvalues only, and rather than factoring out the complete $N+1 \times N+1$ Vandermonde determinant, we should split off the $N \times N$ determinant only. As a result, the hamiltonian H_{N+1} does not reduce to the free hamiltonian H_0 of N non-interacting fermions, but takes the form

$$H(t) = e^{-W(z(t))} H_0 \, e^{W(z(t))} \,, \tag{5.3}$$

with

$$\tilde{W}(z) = \mathrm{Tr} \log(z - \mathbf{T}) \,. \tag{5.4}$$

Note that this object may be thought of as the gauge parameter inducing a background worldvolume gauge field created by the probe. Since the large-N theory is semi-classical, we can simplify (5.3) to

$$H = H_0 + H_1 \,, \qquad H_1 = [H_0, \tilde{W}(z(t))] \,. \tag{5.5}$$

We want to compute the transition amplitude from an initial state $|\mu_F\rangle$, in which the Fermi sea of the N eigenvalues is calm and quiet, to a final state $|\mu_F + \omega\rangle$ with a single excitation with energy ω. In first-order time-dependent perturbation theory, this amplitude takes the form

$$\mathcal{A}(\omega) = \int dt \, \langle \mu_F + \omega | \, [H_0, \tilde{W}(z(t))] \, |\mu_F\rangle = \omega \int dt \, \langle \mu_F + \omega | \, \tilde{W}(z(t)) |\mu_F\rangle \,. \tag{5.6}$$

The matrix operator $W(z)$ is related via a Laplace transform to the "macroscopic loop operator" $W(\ell)$ that creates a boundary to the string world sheet with total boundary length ℓ, with a Dirichlet condition[8] at time t:

$$\tilde{W}(z,t) = \int_0^\infty \frac{d\ell}{\ell} \, e^{-z\ell} W(\ell,t) \,, \qquad W(\ell,t) \equiv \mathrm{Tr} \, e^{\ell \, \mathbf{T}(t)} \,. \tag{5.7}$$

The relevant matrix elements of the macroscopic loop operators $W(\ell)$ have been evaluated in [37, 36], with the result

$$\langle \mu_F + \omega | W(\ell,t) | \mu_F \rangle = e^{-i\delta(\omega)} e^{i\omega t} K_{i\omega}(\sqrt{\mu}\, \ell) \tag{5.8}$$

with $\mu = \mu_c - \mu_F$ as before, and where we have included the phase $e^{-i\delta(\omega)}$ as defined in equation (3.11). This phase is not produced by the matrix model itself, but appears as a separate wave-function renormalization, the so-called leg-pole factor, that relates the matrix model states to the continuum closed string tachyon modes (see for example eq. (7.27) of [12]).

[8]We thank N. Seiberg for a discussion on this point.

Combining (5.6) with (5.8) and the integral formula (3.17) gives

$$\mathcal{A}(\omega) = \int dt\, e^{-i\delta(\omega)}\; e^{i\omega t}\, \frac{\cos\left(\pi\omega\, s(t)\right)}{\sinh(\pi\omega)} \tag{5.9}$$

where $s(t)$ parametrizes the probe trajectory via

$$z(t) = \cosh\left(\pi s(t)\right). \tag{5.10}$$

At this point we can already point out a striking correspondence with the boundary states $|B_s\rangle$. Namely, we can write $\mathcal{A}(\omega)$ as [28]

$$\mathcal{A}(\omega) = \int dt [\langle\omega|B_t^D\rangle\langle P|B_{s(t)}\rangle]_{\omega=P} \tag{5.11}$$

where $|B_t^D\rangle$ is a Dirichlet boundary state at time t. Now for the rolling trajectory (5.2) we have

$$\int dt\; \cos(\pi\omega s(t))\; e^{i\omega t} = \int ds\, \rho(s)\; \cos(\pi\omega s) e^{i\omega t(s)} \tag{5.12}$$

with

$$\rho(s) = \frac{dt}{ds} = \frac{\pi}{1+e^{2\pi s}} - \frac{\pi}{1+e^{-2\pi s}}. \tag{5.13}$$

We can view this quantity ρ as the semi-classical expression of the quantum mechanical probability density $|\psi(s)|^2$ of the probe eigenvalue. It is interesting to note that this expression has poles exactly at the special values of the Liouville boundary state parameter s discussed earlier

$$s = \frac{i}{2}(2m+1) \qquad m \in \mathbf{Z}. \tag{5.14}$$

These values of s correspond to the location $z = 0$ at the top of the inverted harmonic potential. The physical origin of the poles in $\rho(s)$ is of course that the critical trajectory (5.2) has zero velocity at this location, while the periodic recurrence of the poles reflects the periodic orbits of the right-side up harmonic oscillator.

To perform the integral (5.12), we can write $\cos(\pi\omega s) = \frac{1}{2}(e^{i\pi\omega s} + e^{-i\pi\omega s})$ and in each of the two terms we can attempt to close the contour in the upper and lower half complex s-plane respectively. The contributions from the sum over residues gives

$$\sum \text{Res}\, \rho(s)\; \cos(\pi\omega s) \;=\; \frac{\pi}{\sinh(\pi\omega/2)}. \tag{5.15}$$

This contributes a term in production amplitude

$$\mathcal{A}_{\text{Res}}(\omega) = \frac{\pi\, e^{-i\delta(\omega)}\, \lambda^{i\omega t}}{\sinh(\pi\omega/2)\, \sinh(\pi\omega)}, \tag{5.16}$$

which reproduces the continuum production amplitude (3.20) of the rolling tachyon of the $|B_{s=i/2}\rangle$ state. This correspondence, but more accurately the match between (5.11) and (5.9), supports our interpretation of the extra eigenvalue as an unstable D-particle of the 1+1-d string theory. The expression (5.16) omits, however, a long-time divergence arising from the countour at infinity. A precise correspondence involves a somewhat different interpretation of the matrix model computation in terms of boundary states, and is discussed further in [28].

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It is in principle straightforward to compute the higher order corrections to the leading order result (5.16). To find a sensible total production rate, however, one would need to treat the probe quantum mechanically, so that the recoil is included and total energy is preserved. It is evident from the matrix description that all the initial energy (of order $1/\mu$) of the D-particle will be emitted in the form of closed string tachyon radiation.

6. Large N RG formulation of the duality

The matrix model description in section 4 of N D-particles in 1+1-d string theory reveals a universal behavior that depends only on one parameter: the effective string coupling μ (which e.g. sets both the size of the tunneling amplitude (4.14)) as well as of the eigenvalue density $\rho(\mu_F)$ as given in (4.10)). The model itself, however, has two parameters: the number of D-particles N (all assumed to be in their lowest possible energy eigenstate), the (bare) string coupling g_s. We are therefore led to conclude that there should be an equivalence between 1+1-d string theories with different values of N, g_s, but with the same values for μ:

$$(g_s, N) \cong (\tilde{g}_s, \tilde{N}) \qquad \text{if } \mu = \tilde{\mu}\,. \tag{6.1}$$

This is our new proposed duality relation, and can be thought of as a large-N renormalization group transformation that leaves 1+1-d string theory invariant.

A special case of this duality relation is the equivalence of a string theory with N D-particles and coupling g_s and the string without D-particles and string coupling $g_{\text{eff}} = 1/\mu$. This equivalence reduces in the large-N limit to the usual $c = 1$ matrix model duality. As long as N is finite, the string coupling g_s is finite as well, so that the D-particles are not decoupled: they must be described as embedded and interacting with the surrounding closed string theory. The finite N theory thus gives an interesting hybrid description of the effective closed string theory, in which the effective world-sheets are partially made up from open string diagrams and partially from closed string diagrams. Only upon taking the double scaling limit $N \to \infty$, $g_s \to 0$ with μ fixed, can we treat the effective D-particle action (4.1) as a strictly decoupled system. Thus our proposed duality gives a new physical explanation of the origin of the double scaling limit.

Another interesting case is $N = 1$ and $\tilde{N} = 0$. It entails that adding a single D-particle in its lowest energy eigenstate, can be absorbed via a small adjustment of the string coupling. This statement appears to be closely related to the aforementioned result that the critical time-dependent tachyon profile (2.4) with $\lambda = 1/2$ is equivalent to a source free wave-like solution of the closed string background fields. Indeed it is natural to interpret the process of adding an extra D-particle in its lowest energy eigenstate as the quantum description of the classical $\lambda = 1/2$ tachyon bounce. Since we have placed the system in a box by adding a reflecting boundary condition at T_0, the minimal energy bounce will instead take the form of a minimal energy standing wave. This predicts that, on the closed string side of the duality, the $\lambda = 1/2$ wave will also relax to a minimal energy standing wave, which according to our proposal is just a small static shift of the closed string background fields.

To further substantiate this physical equivalence, reconsider the above emission calculation for the special probe trajectory

$$z(t) = 2\lambda\sqrt{\mu}\cosh t \qquad \lambda = \frac{1}{2}\,. \tag{6.2}$$

This trajectory precisely follows the surface of Fermi sea. In terms of the variable $s(t)$ it is described by $\pi s(t) = t$. The corresponding closed string production amplitude is therefore proportional to $\int ds \cos(\pi\omega s)$ and thus vanishes for all non-zero ω. It thus corresponds to a static shift in the closed string background, as advocated. Sub-critical trajectories with $\lambda > 1/2$ are obviously Pauli excluded, while super-critical trajectories with $\lambda < 1/2$ do generate non-trivial emission amplitudes. Eventually, all super-critical trajectories decay to the minimal energy one; in the target-space field theory, the only permanent remnant of the presence of the extra D-particle is a small adjustment of the tachyon background proportional to the associated small shift in the Fermi sea. In the appendix we show how this shift is calculated from the continuum boundary state description of section 3.

It is instructive to consider the dual equivalence (6.1) with $N = \tilde{N} + 1$, which is the smallest renormalization group step. To obtain the explicit form of the corresponding background shift, it turns out to be a bit more practical to first view N as a function of the coupling g_s, whose form is determined via the condition that μ is held fixed. This condition results in a differential equation, which is easily found from (4.8) and (4.5).

Let us summarize. We have formulated a new open/closed string duality relation (6.1) between 1+1-d string theory backgrounds with different numbers of D-particles. This duality provides a new interpretation and physical foundation of the $c = 1$ matrix model, which hopefully will help in putting the string/matrix model duality on a somewhat firmer footing. A problem that remains, however, is that the 1+1-d string theory is non-perturbatively unstable, although we now have a more direct interpretation of this instability as the decay of a D-particle to the "wrong side" of its tachyon barrier, where the potential is unbounded from below. Clearly, it would be of interest to find a consistent completion of 1+1-d string theory, for example by embedding it in a larger self-consistent framework. In the following section we will give a concrete proposal in this direction in terms of IIB superstring theory.

7. $c = 1$ matrix model from IIB string theory

In this section we will consider a dense gas of unstable D-particles in IIB superstring theory, and argue that, in a suitable decoupling limit, its description reduces to the $c = 1$ matrix model.

In comparison to our discussion of D-particles in the 1+1-dimensional bosonic string theory, there are several new ingredients that we need to take into account. First, unstable D-particles in IIB string theory have, besides the open string tachyon mode, also other light degrees of freedom, namely their positions X^0.[9] The non-abelian worldline action of

[9]The worldline theory of an unstable D-particle in type-IIB string theory also contains 32 worldline fermions, whose presence will be ignored in the following discussion.

N particles therefore has a more complicated form [38, 39, 40]

$$S_{DBI} = \int dt\,\mathrm{Tr}\biggl(V(\mathbf{T})\sqrt{\Bigl((1-(D_t\mathbf{T})^2+(D_tX^i)^2\Bigr)\det\left(\delta_{ij}+[X_i,X_j]\right)+ \\ + f(\mathbf{T})[X_i,\mathbf{T}]^2+\dots}\biggr). \tag{7.1}$$

Our goal is to show that, in a suitable high density limit, the tachyon mode T becomes much lighter than the X^i's, so that in this limit the model in fact reduces to the $c=1$ matrix quantum mechanics.

Besides the Born-Infeld action (7.1), the IIB D-particle world-line action also involves a Chern-Simons term that describes its coupling to the RR scalar, the IIB axion field C. To write this term, let us momentarily ignore the other D-particle degrees of freedom, and concentrate on the tachyonic mode only. The complete action, including the Chern-Simons coupling [3, 41, 42], then reduces to (here the $\,'\,$ denotes derivative with respect to T)

$$\begin{aligned} S &= S_{DBI}+S_{CS} \\ &= \int dt\,\mathrm{Tr}\Bigl(V(\mathbf{T})\sqrt{1-(D_t\mathbf{T})^2}\Bigr)+\int dt\,\mathrm{Tr}\Bigl(C\,W'(\mathbf{T})D_t\mathbf{T}\Bigr)\,, \end{aligned} \tag{7.2}$$

where[10]

$$W'(T) = g_sV(T) = \frac{1}{\cosh(T/\sqrt{2})}\,. \tag{7.3}$$

As was recently shown in [17] and [41], this form is completely fixed by the requirement that the known consistent open string tachyon profiles $T(X^0)$ solve the equation of motion of (7.2). Note further that the relation (7.3) between $W(T)$ and $V(T)$ ensures that a D-instanton, which is known to correspond to a trajectory $T(t)$ that runs from the minimum at $T=-\infty$ to the other minimum at $T=+\infty$, carries the correct unit of RR-charge.

Motivated by the preceding discussion of 1+1-d string theory, let us consider a dense gas of M unstable D-particles inside of some finite volume V_9. We wish to study this system in its lowest possible energy state. As we have learned, this means that the tachyon mode T on each D-particle must follow the minimal bounce trajectory (2.4) with $\lambda=\frac{1}{2}$. This minimal trajectory is called an sD-brane in [8], where it was shown that it has the characteristic property that it creates half a unit of flux for the time-derivative of the axion C. M sD-branes inside of a 9-volume V_9 thus produce a flux

$$\int_{V_9} *\,dC = \frac{1}{2}\,M\,. \tag{7.4}$$

[10]Notice that $V(\mathbf{T})$ is identical to its bosonic cousin (2.6) up to a rescaling of T by a factor of $1/\sqrt{2}$. To understand this factor, recall that the intercept in the fermionic string is half that of the bosonic string. In boundary CFT language: to turn on an open string tachyon profile $T(X^0)$, one needs to introduce a boundary interaction of the form

$$S_{bdy} = \int d\xi\,\psi^0(\xi)T'(X^0(\xi))\otimes\sigma_1$$

with σ_1 a Chan-Paton index [2]. Conformal invariance thus requires that $T(X^0)$ has scale dimension 1/2.

Positive and negative M correspond to sD-branes with positive and negative $T(t) = \pm\frac{1}{2}\cosh t$. If we assume that the particles are evenly distributed, we conclude that every particle, via the flux produced by all the other particles, is immersed in a uniform field

$$\partial_0 C = \frac{1}{2}\,\nu\,, \qquad \nu = \frac{M}{V_9}\,. \tag{7.5}$$

How should we incorporate the presence of this background flux into the effective action of the M D-particles? Here we need to be a bit careful. It is tempting to conclude that we need to include a non-zero C in the Chern-Simons term:[11]

$$S_{CS} = \frac{1}{2}\int dt\,\nu\, t\,\mathrm{Tr}\Big(W'(\mathbf{T})D_t\mathbf{T}\Big) = -\frac{1}{2}\int dt\,\nu\,\mathrm{Tr}\,W(\mathbf{T})\,. \tag{7.6}$$

However, since the matrix variables of the non-abelian DBI action include the open string states that stretch between the particles, via the usual open/closed string equivalence it already includes the effect of closed string exchange between the particles! We would therefore be double-counting if we add the CS-term as well.

Now let us instead consider a gas of $N+M$ D-particles inside a small volume V_9, with M large compared to N. Let us choose a localized cluster of N of these particles, and consider them as moving in the closed string background geometry produced by the surrounding gas of M D-particles. The non-abelian DBI action of the N particles now includes the CS-term (7.6). The total tachyon effective potential therefore reads

$$V_{\rm eff}(T) = V(T) - \frac{\nu}{2}\,W(T) \tag{7.7}$$

which for the explicit form of potentials (7.3) reads:

$$V_{\rm eff}(T) = \frac{1}{g_s\cosh(T/\sqrt{2})} \;-\; \nu\sqrt{2}\arctan\Big(\sinh(T\sqrt{2})\Big)\,. \tag{7.8}$$

The second term will only become important at very high densities, when ν is order $1/g_s$. In the following we will assume that there is no fundamental obstruction against preparing the system at such a high density.[12]

The typical form of the effective potential is drawn in figure 2. The effect of the ν term is to raise the left minimum and lower the right minimum of $V_{\rm eff}(T)$. In light of our earlier discussion, it seems natural to interpret this extra term as the rise in the Fermi sea of the tachyon matrix eigenvalues due to the presence of the dense gas of D-particles. In any case, this term has the consequence that tachyon modes that approach from the left, the potential barrier can be made arbitrarily small.

Let us make this explicit. The effective potential $V_{\rm eff}(T)$ has a maximum at

$$T_c = \sqrt{2}\,\mathrm{arctanh}(g_s\nu\sqrt{2})\,, \tag{7.9}$$

[11] Here we are dropping a boundary term in the partial integration.

[12] In any case it is clear that, as a consequence of the scaling limit we are about to take, there is no obstruction from gravitational collapse. The gravitational radius of a region containing M critical D-particles is $\ell_s(\lambda_{\rm 't\ Hooft})^{1/7}$, where the 't Hooft coupling is $\lambda_{\rm 't\ Hooft} = g_s M$. In our double scaling limit, $\lambda_{\rm 't\ Hooft} \to 0$; the string coupling is weaker than in a generic 't Hooft limit. The gravitational curvature is of order the string scale, which in the $g_s \to 0$ limit is much below the Planck scale.

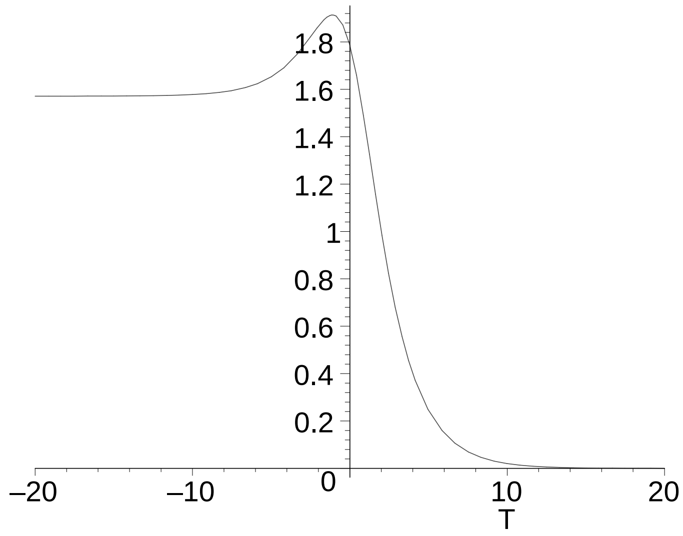

Figure 2: Typical form of the effective tachyon potential $V_{\text{eff}}(T)$ of unstable D-particles in an axion background with $\partial_0 C = \nu$ with ν a constant of order $1/g_s$.

which exists as long as ν is less then a critical value

$$\nu_c = (\sqrt{2}g_s)^{-1}. \tag{7.10}$$

In the limit where ν approaches ν_c

$$\epsilon \equiv 1 - \frac{\nu}{\nu_c} \ll 1\,, \tag{7.11}$$

the effective single particle hamiltonian (obtained after reducing of the matrix quantum mechanics to that of the eigenvalues) near the maximum takes the following form

$$h(T_i) \simeq \alpha \left(\frac{1}{\beta^2}\, e^{\hat{T}_i}\, \hat{\pi}_i^2 + e^{-\hat{T}} - \frac{1}{3} e^{-3\hat{T}} \right) \tag{7.12}$$

and

$$\alpha = \frac{\sqrt{2}\epsilon^{3/2}}{g_s}\,, \qquad \beta = \frac{2\epsilon}{g_s}\,. \tag{7.13}$$

Here we redefined $\hat{T}_i = \frac{1}{\sqrt{2}}(T_i - T_c)$, and $\hat{\pi}_i = \sqrt{2}\pi_i$, so that the new effective potential has its maximum at $\hat{T} = 0$. [We have writen the hamiltonian in the non-relativistic form, which is justified as long as the energy is small compared to $V(T)$. In the new variables, this means that we must restrict to the region in which $e^{\tilde{T}}$ is small compared to β^2/α. As in the previous section, we will put a cut-off $T \leq T_0$ on the tachyon mode, with T_0 small enough to satisfy this condition. In the following, we are going to take the limit $\alpha \to 0$ and $\beta \to \infty$, so this restriction will in fact become irrelevant.]

From the form (7.12) of the effective hamiltonian we deduce that, in this regime with α very small and β very large, the spectrum of h will contain a large number (of order β) of very small eigenvalues (less than α). We would like to use this fact to determine a precise limit in which the T-dynamics decouples from all the other degrees of freedom, and in particular from the space-time motion $X_m(t)$ of the D-particles.

The D-particle motion is governed by the matrix action (omitting factors of order 1)

$$S(X) = \frac{\epsilon^{1/2}}{g_s} \int dt \,\mathrm{Tr}\Big(e^{-\hat{\mathbf{T}}} \{(D_t X_m)^2 + [X_m, \hat{\mathbf{T}}]^2 + [X_m, X_n]^2\}\Big) \,. \tag{7.14}$$

The corresponding hamiltonian reads

$$h(X) = \alpha \,\mathrm{Tr}\Big(\frac{1}{\beta^2}\, e^{\hat{\mathbf{T}}} P_m^2 + \frac{1}{\epsilon}\Big\{ [X_m, \hat{\mathbf{T}}]^2 + [X_m, X_n]^2 \Big\}\Big) \,. \tag{7.15}$$

We would like to obtain an estimate of the ground state energy and of the energy gap of this hamiltonian.

The classical potential in $h(X)$ has flat directions $[X_m, \hat{\mathbf{T}}] = [X_m, X_n] = 0$. These flat directions are well-known to be lifted by quantum corrections. (Recall that the world-line theory of unstable D-particles is not supersymmetric, so there is no cancellation of bosonic vacuum energies.) If we set $\hat{\mathbf{T}} = 0$, a simple scaling argument then shows that the remaining hamiltonian $\tilde{h}(X)$ has a non-zero ground state energy and energy gap proportional to

$$\Delta E \simeq \mathcal{O}\left(\frac{\alpha}{\beta^{4/3}\epsilon^{1/3}}\right) = \mathcal{O}\left(\frac{\alpha\, g_s^{4/3}}{\epsilon^{5/3}}\right) . \tag{7.16}$$

Now, in order to achieve the decoupling of the tachyon mode from the dynamics of the X-degrees of freedom, we would like this energy to be much larger than the effective potential of the single eigenvalue hamiltonian $h(\hat{T}_i)$, which is of order α.

We are now in a position to give a precise characterization of the decoupling limit. We are going to send

$$N \to \infty\,, \qquad g_s \to 0\,, \qquad \epsilon \to 0. \tag{7.17}$$

We keep βN fixed

$$\frac{g_s N}{\epsilon} \qquad \text{fixed}\,. \tag{7.18}$$

Further, we want to make sure that the energy scale ΔE in (7.16) becomes large, so we have

$$labelthree \frac{\epsilon^{5/4}}{g_s} \sim N\epsilon^{1/4} \to 0\,. \tag{7.19}$$

Upon taking this limit, the dynamics of the N D-particles reduces to the $c = 1$ matrix quantum mechanics of the tachyon mode $\hat{\mathbf{T}}$. Note that the limit in particular involves sending the string coupling to zero (even fast enough so that $g_s N \to 0$), so that the D-particles indeed decouple from the IIB closed strings in the bulk.

8. Conclusions

Our new proposal is that 1+1-dimensional string theory, via its equivalence with the $c = 1$ matrix model, can be identified with the above-defined decoupling limit of a dense collection of unstable D-particles in IIB string theory. The evidence supporting this identification is twofold: (i) the worldline theory of the N unstable D-particles in this limit reduces to the $c = 1$ matrix model, and (ii) the interaction of the matrix model degrees of freedom with the 1+1-d closed strings is consistent with their interpretation as the tachyon field of the corresponding D-particles. We will now make some comments about this duality.

(i) A perhaps somewhat surprising aspect of the proposed duality is that it involves a dimensional reduction from a 10-d to a 2-d string theory. This dimensional reduction amounts to the statement that in the near-horizon limit only the s-wave sector of the IIB string theory survives. A somewhat schematic explanation for how this may come about is as follows. Consider the full 10-d background produced by a dense collection of many unstable IIB D-particles. This background has SO(9) symmetry, and is thus naturally describes in polar coordinates (x^0, r, Ω). Since the boundary state of the D-particles contains a tadpole for the dilaton and graviton, the worldsheet action of a string moving in this background will take the general form (omitting worldsheet fermions)

$$S_{ws} = \frac{1}{4\pi} \int d^2\sigma \left(-\mathrm{A}(r)(\partial_a x^0)^2 + (\partial_a r)^2 + \Phi(r) R^{(2)} + \mathrm{B}(r)(\partial_a \Omega)^2 \right). \tag{8.1}$$

Now because of the non-trivial radial dependence of the dilaton field $\Phi(r)$, the radial coordinate r acquires a non-trivial transformation under worldsheet scale transformations $z \to (1 + \delta\epsilon)z$:

$$\delta r = \delta\epsilon \nabla_r \Phi(r)\,. \tag{8.2}$$

Thus if this gradient gets large, we can interpret the radial evolution as a worldsheet renormalization group flow: large values of r correspond to the ultra-violet, and small values of r to the infra-red region of the worldsheet CFT. Now if we freeze r, the angular part of the worldsheet CFT reduces to an O(9) sigma-model with a non-trivial RG flow: it is expected to become strongly coupled and develop a mass-gap in the IR. In geometrical terms, this means that the S^8 shrinks to zero size, leaving behind only the s-wave modes. Unlike their supersymmetric IIA cousins, the IIB D-particles do not produce a stabilizing flux through the S^8 that would prevent it from collapsing. Assuming that the remaining radial and time coordinates remain massless, their sigma-model action must be characterized by a solution to the 2-d target space equation of motion of (3.2).

(ii) It is natural to ask whether there are lessons from all this about 26-dimensional critical string theory. If our identification between the closed-string tachyon with the density of D-particles persists in the higher dimensional theory, it provides additional support for the long-standing suspicion that the endpoint of the closed bosonic string tachyon condensation may be described by 1+1-d non-critical string theory [43].

(iii) The strategy followed in section 7 may possibly be generalized to construct decoupling limits of many other unstable D-brane systems, while reducing their world volume theory to matrix quantum mechanics. The general idea is as follows.

Unstable D-branes generally arise as D-sphalerons, the minimal energy configuration at the half-way point of a euclidean D-instanton trajectory. The D-instantons couple to some specific RR-form, C_{RR}, which acts as the corresponding theta-angle. By turning on a time-like gradient for C_{RR} one can in effect introduce a chemical potential that drastically reduces the height of the sphaleron barrier, thus making the unstable D-branes very light. By considering a suitable large-N limit, one may thus hope to isolate a single matrix valued sphaleron mode.

As a concrete example of this procedure, consider Yang-Mills gauge theory on $S^3 \times R$. This theory has sphaleron configurations, that for $\mathcal{N} = 4$ SYM are related via AdS/CFT duality to unstable D-particles in AdS [44]. We can turn on a chemical potential that makes the sphaleron mode light by introducing a time-dependent theta-angle $\theta = \nu t$. This results in an extra contribution to the SYM hamiltonian proportional to ν times the Chern-Simons number $CS = \int \mathrm{Tr}(AdA + \frac{2}{3}A^3)$. (Such a modification of the hamiltonian naturally arises if one couples the gauge theory to chiral fermions and considers the model at finite fermion density. The parameter ν then corresponds to the chemical potential for the anomalous fermion number Q_f.) In the limit where ν approaches a critical value ν_c, the hamiltonian for the (appropriately defined [45]) constant gauge field modes reduces to

$$H \simeq \mathrm{Tr}\left(E_i^2 + \epsilon A_i A_i + g\epsilon^{ijk} A_i [A_j, A_k]\right) \tag{8.3}$$

with $\epsilon = \nu_c - \nu \to 0$. In this limit, the constant gauge field modes decouple from all the other modes. This therefore defines a matrix model reduction of gauge theory. The matrix potential has one unstable sphaleron mode, interacting with two stable modes.

(iv) The identification of 1+1-d closed strings, as excitations of the Fermi sea of eigenvalues of the non-abelian open string tachyon, opens up a very interesting new perspective on the interactions between D-particles and strings. Whereas small fluctuations of the Fermi sea have a direct perturbative closed string interpretation, larger non-linear excitations may generate configurations that can only be given a non-perturbative interpretation. There are two distinct kinds of large nonlinearities on the Fermi sea: large gradients and large amplitudes.

When an incoming pulse has a spatial gradient above a certain threshold, it will lead to a cresting, or folding, of the Fermi surface [46, 13, 47]. This phenomenon, which has always been a somewhat puzzling feature of the $c = 1$ matrix model, now gets a rather natural interpretation as the creation of an unstable D-particle from closed strings. The lifetime of the D-particle excitation grows logarithmically with the inverse distance between the top of the wave and the top of the potential. Eventually it decays into closed string radiation. At intermediate times, however, it should reveal the presence of on-shell *open* string excitations, which in the matrix model correspond to excitations of size $\sqrt{\hbar}$ rather than $\hbar$.

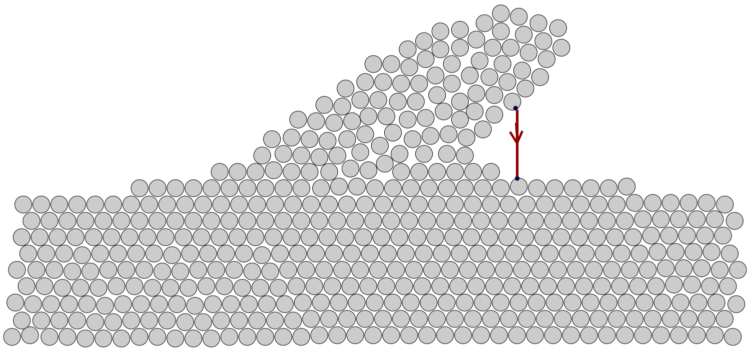

Figure 3: Cartoon of open string creation during cresting of the fermi surface. Each circle represents an eigenvalue occupying phase-space area $\hbar$. The open string depicted naturally has excitations of energy $\sqrt{\hbar}$.

The other kind of strong nonlinearity involves large amplitudes. Again there is a threshold, which is when the pulse extends above the top of the potential barrier. In this case, there will be tearing of the Fermi surface as the top of the wave is sucked into the gorge of eternal peril. Luckily, via our new physical interpretation, we can now clarify what happens to the part of the wave that is lost: it corresponds to a collection of IIB D-particles that decays via a tachyon that rolls down towards the right-hand side of the effective potential in figure 2. The same fate awaits any eigenvalue that penetrates the potential barrier via quantum mechanical tunneling. The near-horizon $c = 1$ string theory is therefore not a completely decoupled theory, but nonetheless — when thought of as embedded inside of the IIB string theory — gives a completely adequate description of the physics of the meta-stable bound state of D-particles.[13]

It is natural to look for nonlinear phenomena in the matrix model that can be interpreted as black hole formation in the 1+1-d effective theory [43]. It will be interesting to revisit the matrix black hole construction of [48], which can be interpreted as turning on a Wilson line of the D-particle world-line gauge field.

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[13] One could perhaps try, however, to obtain a self-consistent matrix model in which the Fermi sea is filled on both sides of the potential barrier [33]. In the IIB string theory, this would need to be interpreted as a dense gas of both sD-branes and anti-sD-banes, with positive and negative (7.4) respectively.

A. Closed-string backreaction of the rolling tachyon

In this appendix we will compute the closed string tachyon state produced by a rolling open string tachyon on a single D-particle. We will do the calculation for the critical bounce trajectory (2.4) with $\lambda = \frac{1}{2}$, though our result is easily generalized to arbitrary $\lambda \leq 1/2$. We will find that the rolling tachyon produces, apart from a discrete series of interesting transients, also a *static* shift in the closed string tachyon, which needs to be interpreted as a small shift in μ. The result will be in accord with our proposed duality between the D-particles and the $c = 1$ matrix eigenvalues. We will also see that the $1 + 1$-d string theory gives an extremely clean example of the paradigm for describing closed strings as imaginary D-branes, recently advocated in [8, 9, 10].

We will extract the response of the closed-string tachyon to the D-particle by factorizing the annulus diagram onto on-shell closed-string poles. We begin by considering the annulus amplitude

$$\mathcal{A}_{s_1,s_2}(\omega) = \int_0^1 d\tilde{q}\, \langle B_{s_2}| \otimes \langle\langle \omega|\ \tilde{q}^{L_0} |B_\lambda\rangle \otimes |B_{s_1}\rangle \cdot \mathcal{A}_{\rm ghost}\,. \tag{A.1}$$

Here $\langle\langle\omega|$ is the Ishibashi state built on the primary $\langle 0|\, e^{-i\omega X^0}$ of the X^0 CFT. Note that we have already performed the integral over the phase of the closed string modular parameter $\tilde{q}$, which implements level-matching, $L_0 = \bar{L}_0$. The ghost part of the annulus is

$$\mathcal{A}_{\rm ghost} = \frac{\eta^2(\tilde{q})}{\tilde{q}}\,. \tag{A.2}$$

The X^0 piece of this amplitude is

$$\langle\langle\omega|\ \tilde{q}^{L_0}|B_\lambda\rangle = \frac{\langle e^{-i\omega X^0}\rangle}{\eta(\tilde{q})} = \frac{\pi\, \tilde{q}^{-\omega^2}}{\eta(\tilde{q})\sinh \pi\omega} \tag{A.3}$$

where the expectation value denotes the one-point function on the disk. Finally, the Liouville part of this amplitude is [21]

$$\int_{\mathcal{C}} \frac{dP}{2\pi i}\, \frac{\tilde{q}^{P^2}}{\eta(\tilde{q})}\ \langle B_{s_2}|v_P\rangle\, \langle v_P|B_{s_1}\rangle \tag{A.4}$$

where the matrix elements are as given in (3.10). The contour of integration is $\mathcal{C} = -i\frac{Q}{2} + \mathbb{R}$.

Putting things together, we find that all factors $\eta(\tilde{q})$ cancel, leaving a trivial integration over the modular parameter $\tilde{q}$. It reduces to the massless propagator

$$\int_0^1 \frac{d\tilde{q}}{\tilde{q}}\ \tilde{q}^{P^2-\omega^2} = \frac{1}{P^2-\omega^2}\,. \tag{A.5}$$

To extract the resulting on-shell background, we follow the general prescription derived in [10] and take the discontinuity in ω of this object:

$$\frac{1}{P^2-\omega^2} \longrightarrow i\pi\frac{\delta(P-\omega)}{\omega}\,, \tag{A.6}$$

so that we can use the delta-function to do the integral over P. Collecting all these facts, we have

$$\mathcal{A}_{s_1,s_2}(\omega) = \tilde{c}\frac{i\pi\cos(\pi s_2\omega)\cos(\pi s_1\omega)}{(\sinh\pi\omega)^3}, \tag{A.7}$$

Now let us specialize the boundary states to fit the physical problem we wish to study. First we set $s_1 = i/2$ so that the state $|B_{s_1}\rangle$ represents the D-particle. The other state we decompose as

$$\langle B_{s_2}| = \int \frac{d\ell}{\ell}\, e^{-\ell\sqrt{\mu}\cosh\pi s_2}\,\langle W(\ell)|\,. \tag{A.8}$$

The states $\langle W(\ell)|$ we interpret as the position eigenstate for the tachyon mode via the identification $\ell = e^{\varphi}$. Using once again the integral formula (3.17), we can thus write the amplitude as

$$\mathcal{A}_{s_1,s_2} = i\tilde{c}\int_0^\infty \frac{d\ell}{\ell}\, e^{-\ell\sqrt{\mu}\cosh\pi s_2}\frac{\pi K_{i\omega}(\sqrt{\mu}\,\ell)}{\sinh(\pi\omega)\,\sinh(\pi\omega/2)} \tag{A.9}$$

The position-space profile of the shift in the closed string tachyon is therefore

$$\delta\mathcal{T}(\ell,\omega) = \tilde{c}\frac{\pi\,\ell^2\,K_{i\omega}(\sqrt{\mu}\,\ell)}{\sinh(\pi\omega)\,\sinh(\pi\omega/2)}\,. \tag{A.10}$$

To extract the time-dependence of this background, we Fourier transform using the contour in the figure; the integral may be done by residues.

$$\delta\mathcal{T}(\ell,t) = \int\frac{d\omega}{2\pi}\,e^{i\omega t}\,\delta\mathcal{T}(\ell,\omega) = \delta\mathcal{T}(\ell) + \sum_{n=1}^{\infty} e^{-nt}c_n(\ell) \tag{A.11}$$

The terms with finite n are transients which represent the splash of the probe D-particle into the Fermi sea. The momenta of the transients are quantized in units of the frequency of the harmonic oscillator appearing in the euclidean continuation of the matrix quantum mechanics. The distinction between odd and even multiples of this basic frequency is the distinction between lengths and laps under the barrier.

The static piece of the shift in the tachyon background is (with $\ell = e^{\varphi}$)

$$\delta\mathcal{T}(\varphi) \propto \ell^2\,\partial_\nu K_\nu(\sqrt{\mu}\,\ell)|_{\nu=0} = \ell^2 I_0(\sqrt{\mu}\ell)\,. \tag{A.12}$$

which in the asymptotic region amounts to a shift $\delta\mathcal{T}(\varphi) \propto e^{2\varphi}$. Hence the shift at the location $\varphi = 0$ of the "dilaton wall" is of order one. Since $\mathcal{T}(0) \simeq \frac{\mu}{2}\log\mu$, we find that the presence of the extra D-particle amounts to a shift $\delta\mu$ of order

$$\delta\mu \propto (\log\mu)^{-1}, \tag{A.13}$$

in accordance with the characteristic level density $\rho(\mu) = \frac{\partial N}{\partial\mu} \simeq -\frac{2}{\pi}\log\mu$ of the $c = 1$ matrix model.

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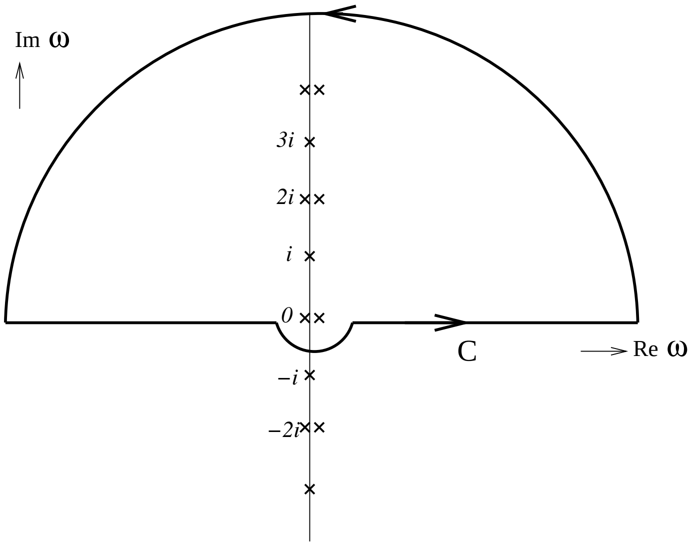

Figure 4: The denominator $\sinh \pi\omega \ \sinh \frac{\pi\omega}{2} = \pi^2\omega^2 \prod_{n=1}^{\infty} \left(\left(\frac{\omega}{2n}\right)^2 + 1 \right)^2 \left(\left(\frac{\omega}{2n+1}\right)^2 + 1 \right)$ has double zeros at even imaginary integers and single zeros at odd imaginary integers. The physical tachyon response is obtained by Fourier transforming using this contour.

References

[1] P. Yi, *Membranes from five-branes and fundamental strings from Dp-branes*, *Nucl. Phys.* **B 550** (1999) 214 [hep-th/9901159];
O. Bergman, K. Hori and P. Yi, *Confinement on the brane*, *Nucl. Phys.* **B 580** (2000) 289 [hep-th/0002223].

[2] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[3] A. Sen, *Rolling tachyon*, *J. High Energy Phys.* **04** (2002) 048 [hep-th/0203211]; *Tachyon matter*, *J. High Energy Phys.* **07** (2002) 065 [hep-th/0203265]; *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[4] A. Sen, *Time evolution in open string theory*, *J. High Energy Phys.* **10** (2002) 003 [hep-th/0207105]; *Time and tachyon*, *Int. J. Mod. Phys.* **A 18** (2003) 4869 [hep-th/0209122].

[5] G. Gibbons, K. Hashimoto and P. Yi, *Tachyon condensates, carrollian contraction of Lorentz group and fundamental strings*, *J. High Energy Phys.* **09** (2002) 061 [hep-th/0209034];
G.W. Gibbons, K. Hori and P. Yi, *String fluid from unstable D-branes*, *Nucl. Phys.* **B 596** (2001) 136 [hep-th/0009061].

[6] M. Gutperle and A. Strominger, *Spacelike branes*, *J. High Energy Phys.* **04** (2002) 018 [hep-th/0202210];
A. Strominger, *Open string creation by S-branes*, hep-th/0209090.

[7] M. Gutperle and A. Strominger, *Timelike boundary Liouville theory*, *Phys. Rev.* **D 67** (2003) 126002 [hep-th/0301038].

[8] A. Maloney, A. Strominger and X. Yin, *S-brane thermodynamics*, *J. High Energy Phys.* **10** (2003) 048 [hep-th/0302146].

JHEP12(2003)054

[9] N. Lambert, H. Liu and J. Maldacena, *Closed strings from decaying D-branes*, hep-th/0303139.

[10] D. Gaiotto, N. Itzhaki and L. Rastelli, *Closed strings as imaginary D-branes*, hep-th/0304192.

[11] P. Ginsparg and G.W. Moore, *Lectures on 2-D gravity and 2-D string theory*, hep-th/9304011.

[12] I.R. Klebanov, *String theory in two-dimensions*, hep-th/9108019.

[13] J. Polchinski, *What is string theory?*, hep-th/9411028.

[14] A. Wachowski and L. Wachowski, *The matrix: reloaded*, to appear.

[15] J. Polchinski, *On the nonperturbative consistency of $D = 2$ string theory*, *Phys. Rev. Lett.* **74** (1995) 638 [hep-th/9409168].

[16] A.M. Polyakov, *Gauge fields and space-time*, *Int. J. Mod. Phys.* **A17S1** (2002) 119–136 [hep-th/0110196].

[17] D. Kutasov and V. Niarchos, *Tachyon effective actions in open string theory*, *Nucl. Phys.* **B 666** (2003) 56 [hep-th/0304045].

[18] N. Seiberg, *Notes on quantum Liouville theory and quantum gravity*, *Prog. Theor. Phys. Suppl.* **102** (1990) 319.

[19] J. Polchinski, *Remarks on the Liouville field theory*, *Presented at strings '90 conf.*, College Station, Mar 12-17, 1990.

[20] V. Fateev, A.B. Zamolodchikov and A.B. Zamolodchikov, *Boundary Liouville field theory. I: boundary state and boundary two-point function*, hep-th/0001012.

[21] J. Teschner, *Remarks on liouville theory with boundary*, hep-th/0009138.

[22] A. Rajaraman and M. Rozali, *D-branes in linear dilaton backgrounds*, *J. High Energy Phys.* **12** (1999) 005 [hep-th/9909017].

[23] B. Ponsot and J. Teschner, *Boundary Liouville field theory: boundary three point function*, *Nucl. Phys.* **B 622** (2002) 309 [hep-th/0110244].

[24] I.K. Kostov, *Boundary correlators in 2D quantum gravity: Liouville versus discrete approach*, *Nucl. Phys.* **B 658** (2003) 397 [hep-th/0212194].

[25] A.B. Zamolodchikov and A.B. Zamolodchikov, *Liouville field theory on a pseudosphere*, hep-th/0101152.

[26] E.J. Martinec, *The annular report on non-critical string theory*, hep-th/0305148.

[27] I.R. Klebanov, J. Maldacena and N. Seiberg, *D-brane decay in two-dimensional string theory*, *J. High Energy Phys.* **07** (2003) 045 [hep-th/0305159].

[28] J. McGreevy, J. Teschner and H. Verlinde, *Classical and quantum D-branes in 2d string theory*, hep-th/0305194.

[29] E. Brezin, C. Itzykson, G. Parisi and J.B. Zuber, *Planar diagrams*, *Commun. Math. Phys.* **59** (1978) 35.

[30] S.R. Das and A. Jevicki, *String field theory and physical interpretation of $D = 1$ strings*, *Mod. Phys. Lett.* **A 5** (1990) 1639.

JHEP12(2003)054

[31] S.H. Shenker, *The strength of nonperturbative effects in string theory*, *Presented at the Cargese workshop on random surfaces, quantum gravity and strings*, Cargese, France, May 28 – Jun 1, 1990, RU-90-47.

[32] J. Polchinski, *Combinatorics of boundaries in string theory*, *Phys. Rev.* **D 50** (1994) 6041 [hep-th/9407031].

[33] A. Dhar, G. Mandal and S.R. Wadia, *Discrete state moduli of string theory from the $C = 1$ matrix model*, *Nucl. Phys.* **B 454** (1995) 541 [hep-th/9507041].

[34] Z. Yang, *Fermion coupling and dynamical loops in $D = 1$ random matrix models*, UTTG-20-90.

[35] J.A. Minahan, *Matrix models and one-dimensional open string theory*, *Int. J. Mod. Phys.* **A 8** (1993) 3599 [hep-th/9204013].

[36] V.A. Kazakov and I.K. Kostov, *Loop gas model for open strings*, *Nucl. Phys.* **B 386** (1992) 520 [hep-th/9205059].

[37] G.W. Moore, N. Seiberg and M. Staudacher, *From loops to states in 2-D quantum gravity*, *Nucl. Phys.* **B 362** (1991) 665.

[38] M.R. Garousi, *Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action*, *Nucl. Phys.* **B 584** (2000) 284 [hep-th/0003122].

[39] J. Kluson, *Proposal for non-BPS D-brane action*, *Phys. Rev.* **D 62** (2000) 126003 [hep-th/0004106].

[40] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras and S. Panda, *T-duality and actions for non-BPS D-branes*, *J. High Energy Phys.* **05** (2000) 009 [hep-th/0003221].

[41] K. Okuyama, *Wess-Zumino term in tachyon effective action*, *J. High Energy Phys.* **05** (2003) 005 [hep-th/0304108].

[42] A. Sen, *Dirac- Born-Infeld action on the tachyon kink and vortex*, *Phys. Rev.* **D 68** (2003) 066008 [hep-th/0303057].

[43] Work in progress.

[44] N. Drukker, D.J. Gross and N. Itzhaki, *Sphalerons, merons and unstable branes in AdS*, *Phys. Rev.* **D 62** (2000) 086007 [hep-th/0004131].

[45] P. van Baal and N.D. Hari Dass, *The theta dependence beyond steepest descent*, *Nucl. Phys.* **B 385** (1992) 185;
B. van den Heuvel, *Glueballs on the three-sphere*, *Nucl. Phys.* **B 488** (1997) 282 [hep-lat/9608101].

[46] J. Polchinski, *Classical limit of (1+1)-dimensional string theory*, *Nucl. Phys.* **B 362** (1991) 125;
M. Natsuume and J. Polchinski, *Gravitational scattering in the $C = 1$ matrix model*, *Nucl. Phys.* **B 424** (1994) 137 [hep-th/9402156].

[47] S.R. Das and S.D. Mathur, *Folds, bosonization and nontriviality of the classical limit of 2-d string theory*, *Phys. Lett.* **B 365** (1996) 79 [hep-th/9507141].

[48] V. Kazakov, I.K. Kostov and D. Kutasov, *A matrix model for the two-dimensional black hole*, *Nucl. Phys.* **B 622** (2002) 141 [hep-th/0101011].

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